Exam is closed books, closed notes, no electronic devices are allowed. Do not bring laptops, tablets, apple-watches, calculators (if you bring them they must be inside of your bag). Cellphones must be turned off and inside of your bags. Your bags must be placed next to the walls. You cannot leave the room before you turn in the exam paper. Your UNT ID card may be asked to be provided during the exam. You only need your pen and ID card. You should write your answers on the space provided on the exam paper. You will be graded for accuracy, completeness, and clarity.

1. (20 pts.) Solve $T(n) = 4T\left(\frac{n}{2}\right) + n^2$, $T(1) = 1$ You can assume that $n = 2^k$. 
2. (20 pts.) What is the answer to the following recurrence relations. (Give the answer as \( \Theta \); no justification is necessary. You may assume that \( T(1) = 1 \)).

(a) \( T(n) = 8T\left(\frac{n}{2}\right) + n^2, \ n = 2^k \)

(b) \( T(n) = 2T\left(\frac{n}{2}\right) + n, \ n = 2^k \)

(c) \( T(n) = T(n - 1) + 1 \)

(d) \( T(n) = 2T(n - 2) + 1 \)

3. (10 pts.) What is the time complexity of the following program? Justify your answer, by writing a brief narrative. You can assume that \( N \) is an integer.

\[
\begin{align*}
\text{Read}(N) \\
X \gets N^2 \\
\text{While } X > 1 \text{ do} \\
S \gets 0 \\
\text{For } i \gets 1 \text{ to } X \text{ do} \\
S \gets S + i \\
\text{EndFor} \\
X \gets X \ (DIV) 2 \\
\text{EndWhile} \\
\text{End.}
\end{align*}
\]
4. (20 pts.) Circle True or False (no justification is needed):

(a)- \( f(n) = O(g(n)) \) implies that \( g(n) = \Omega(f(n)) \)  True, False

(b)- Any acyclic undirected graph on \( n \) vertices has \( n - 1 \) edges.  True, False

(c)- If \( \log(f(n)) = \Theta(\log(g(n))) \), then \( f(n) = \Theta(g(n)) \)  True, False

(d)- Chromatic Number of a graph is always \( \leq \) the minimum degree + 1.  True, False

(e)- Let \( G \) be a Tree, on \( n \) vertices, then, the sum of degrees in \( G \) equals \( n - 1 \).  True, False

5. (20 pts.) Circle True or False (no justification is needed):

(a)- Insertion takes \( O(\log(n)) \) time, in the worst case, in a binary search tree.  True, False

(b)- Deletion of an item in the middle of a priority queue (heap) can be done in \( O(\log(n)) \) time in the worst case.  True, False

(c)- Finding the smallest element in a min heap can be done in \( O(1) \) time.  True, False

(d)- Finding the smallest element in a binary search tree on \( n \) elements can be done in \( O(\log(n)) \) time in the worst case.  True, False

(e)- If the post order traversals of two binary trees are identical, then the trees are identical. Two Binary trees are identical, if they have the same shape and the same contents.  True, False

6. (10 pts) Suppose that we have \( n \) in the range of 1 to 1000, and we want to sort them. Name a sorting algorithm that sorts these integers in order of \( O(n) \) (No justification is needed).