Order the following functions by growth rate: $N$, $\sqrt{N}$, $N^{1.5}$, $N^2$, $N \log N$, $N \log \log N$, $N \log^2 N$, $N \log(N^2)$, $2/N$, $2^N$, $2^{N/2}$, 37, $N^2 \log N$, $N^3$. Indicate which functions grow at the same rate.

Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$. Which of the following are true?

a. $T_1(N) + T_2(N) = O(f(N))$

b. $T_1(N) - T_2(N) = o(f(N))$

c. $\frac{T_1(N)}{T_2(N)} = O(1)$

d. $T_1(N) = O(T_2(N))$

Prove that for any constant $k$, $\log^k N = o(N)$.

Find two functions $f(N)$ and $g(N)$ such that neither $f(N) = O(g(N))$ nor $g(N) = O(f(N))$.

For the following program fragment:

Give an analysis of the running time (Big-Oh will do).
6) For the following program fragment:

```c
sum = 0;
for( i = 0; i < n; ++i )
    for( j = 0; j < i; ++j )
        ++sum;
```

Give an analysis of the running time (Big-Oh will do).

```c
sum = 0;
for( i = 0; i < n; ++i )
    for( j = 0; j < i * i; ++j )
        for( k = 0; k < j; ++k )
            ++sum;
```

7) An algorithm takes 0.5 ms for input size 100. How long will it take for input size 500 if the running time is the following (assume low-order terms are negligible)?
   a. linear
   b. $O(N \log N)$
   c. quadratic
   d. cubic

8) Solve
   $T(n)=4T(n/4)+n$
   $n=4^k$, $T(1)=1$

9) Solve
   $T(n)=4T(n/2)+n$
   $n=2^k$, $T(1)=1$