CSCE4110- Fall 2017
Hw-1 Solutions.

1. Solution is provided in the recurrence relation material posted on the class page.
   \[ T(n) = \Theta(n^3). \]

2. \[ T(n) = 16T\left(\frac{n}{4}\right) + n^2 \]
   \[ = 16\left[16T\left(\frac{n}{4}\right) + \left(\frac{n}{4}\right)^2\right] + n^2 \]
   \[ = 16^2T\left(\frac{n}{16}\right) + 2n^2 \]
   \[ = 16^3T\left(\frac{n}{32}\right) + 3n^2 \]
   After k steps, we can see a pattern: \[ 16^kT\left(\frac{n}{4^k}\right) + kn^2 \]
   Assume \( n = 4^k \), that implies we need \( (k = \log_4(n)) \) substitution to get down
   the base case.
   \[ T(n) = cn^2 + n^2 \log_4(n) \]
   \[ = \Theta(n^2 \log(n)) \]
   
   Remark: Assume \( T(1) = c \)

3. \[ T(n) = 7T\left(\frac{n}{3}\right) + n^2 \]
   \[ = 7\left[7T\left(\frac{n}{3}\right) + \left(\frac{n}{3}\right)^2\right] + n^2 \]
   \[ = 7^2T\left(\frac{n}{9}\right) + n^2 + 7\frac{n^2}{9} \]
   \[ = 7^3T\left(\frac{n}{27}\right) + n^2 + \frac{7}{9}n^2 + n^2\left(\frac{7}{9}\right)^2 \]
   After k steps, we can see a pattern: \[ 7^kT\left(\frac{n}{3^k}\right) + n^2(\sum_{i=0}^{k-1} \left(\frac{7}{9}\right)^i) \]
   Assume \( n = 3^k \)

   Note that \[ \sum_{i=0}^{k-1} \left(\frac{7}{9}\right)^i = \frac{\left(\frac{7}{9}\right)^k - 1}{\frac{7}{9} - 1} = \frac{2}{9} \left( 1 - \left(\frac{7}{9}\right)^k \right) = \frac{2}{9} \left( 1 - \frac{7^{\log n}}{n^2} \right) = \frac{2}{9} \left( 1 - \frac{n^{\log 7}}{n^2} \right) \]

   Thus \[ T(n) = n^{\log 7}T(1) + \frac{2}{9}n^2 \left( 1 - \frac{n^{\log 7}}{n^2} \right) \]
   \[ = n^{\log 7} + \frac{2}{9}n^2 - \frac{2}{9} n^{\log 7} \]
   \[ = \frac{2}{9}n^2 + \frac{2}{9} n^{\log 7} \]
   \[ = \Theta(n^2). \]

4. \[ T(n) = 7T\left(\frac{n}{2}\right) + n^2 \]
   Was Solved in class

5. \[ T(n) = T(n - 1) + n^2 \]
   A similar problem is solved in Hw-2 solutions.

6. Was Solved in class.