FLEXIBLE ALLOCATION OF CAPACITY IN
MULTI-CELL CDMA NETWORKS

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Abstract - The effect of reverse power levels on the
capacity of a code-division multiple-access (CDMA) cellular
network is evaluated. The inter-cell and
intra-cell interferences of every cell on every other
are first calculated for a given network topology.
Based on this, the nominal power of users is
increased by a factor we call the Power Compensation
Factor (PCF) which enables small cells to over-
come the excessive interference from adjacent large
cells. The PCF’s provide flexibility in the allo-
cation of capacity. By changing the PCF’s, capacity
can be exchanged between cells. The implied cost
of the network capacity function with respect to the
PCF’s is calculated and used to maximize the cap-
acity of the network by applying a gradient descent
algorithm.

I. INTRODUCTION

The outage probability of calls within a single cell
in a cellular code-division multiple access (CDMA)
network depends on the interference of users within
that cell (intra-cell interference) and on the inter-
ference of users in adjacent cells (inter-cell interfer-
ence) [1],[2]. Thus, the number of simultaneous calls
that can be handled within one cell depends on the
number of simultaneous calls in neighboring cells.
Since CDMA is interference limited, any decrease in
the amount of interference translates into a capacity

In a network with large cells adjacent to small cells,
users at the boundaries of large cells cause a lot of in-
ference to users in small cells thereby reducing the
capacity of small cells [3]. To alleviate this problem,
we propose to adjust the nominal power of the users
in every cell by a factor we call the Power Compensation
Factor (PCF). We then evaluate the capacity of the
entire network as a function of all the PCF’s and
present an optimization framework which allows us
to maximize the capacity. In order to quantify the
trade-off of capacity, we calculate the implied costs
[4] of the capacity region as a function of the PCF’s.
Thus, the power compensation factor turns out to be
an effective control parameter which the network
administrator can use to flexibly distribute capacity
in a multi-cell CDMA network.

II. INTER-CELL INTERFERENCE

Basic Model: Consider two cells i and j. We assume
that each user is always communicating through the
nearest base station and is power controlled by that
base station. A user A located at coordinates (x, y)
and served by the base station of cell j, is at distance
r_j(x, y) from base station j, and distance r_i(x, y)
from base station i. Let n_j be the number of users
in cell j. We define the user density of this cell, \rho_j,
as:
\[ \rho_j = \frac{n_j}{\text{Area}_j} \]
where Area_j is the area of cell j. It is assumed
that the power control mechanism overcomes both
large scale path loss and shadow fading. It does not,
however, overcome the fast fluctuations of the signal
power associated with Rayleigh fading. The propa-
gation loss of user A to cell j is modeled as the prod-
uct of the mth power of distance and a log-normal
component representing shadowing losses. Now let
X_i denote the Rayleigh random variable that repre-
sents the fading on the path from this user to cell i
[5]. The average of X_i^2 is the log-normal fading on
that path, i.e., \[ E[X_i^2] = 10^{-\zeta_i/10} \]
where \zeta_i is the decibel attenuation due to shadowing, and has zero
mean and standard deviation \sigma_i. Consequently, the relative average interference at cell i caused by all
users in cell j is given by [3]:
\[ I_{ij} = E \int \int_{\text{cell}_j} \frac{r_j^m(x, y)10^{\zeta_i/10}}{r_i^m(x, y)/X_i^2} \rho_j dA(x, y). \]
Figure 1: Inter-cell interference on cell $i$ when users are (a) in cell $i$ or cell $j$ and communicating with cell $j$, (b) in cell $i$ or cell $k$ and communicating with cell $k$, (c) in cell $j$ or cell $k$ and communicating with either cell $j$ or $k$.

**Soft Handoff Model:** In order to study the performance of a soft handoff system, we assume that each user communicates with either the closest or the second closest base station [1]. Consider the three cell network given in Fig. 1. The shaded regions shown in part (a) are the regions wherein users can be in soft handoff only with base stations $i$ and $j$. Thus, a user in either region will introduce inter-cell interference to base station $i$ if it communicating with base station $j$. But this happens only if the propagation loss to base station $j$ is less than that to base station $i$, in which case the user is power-controlled by $j$. Thus, the interference to base station $i$ from users in the shaded regions given in Fig. 1 part (a) is:

$$I_{ji} = \int_{\text{shaded-regions (a)}} \frac{r_i^m(x,y)}{r_j^m(x,y)} \cdot \frac{E_b(10\gamma_i^{10})}{10^{10}} \cdot X_i^2$$

$$r_j^m(x,y)10^{10} \leq r_i^m(x,y)10^{10} |\rho d A(x,y)|. \quad (3)$$

A similar calculation follows for the shaded regions shown in part (b). For the shaded regions shown in part (c), the users can be in soft handoff only with cells $j$ and $k$. In either case, the users there will cause inter-cell interference to cell $i$. Thus we can calculate the inter-cell interference of every user on cell $i$ with a calculation similar to that of equation (3). The calculations extend to a general network where each user communicates with either of the two closest base stations.

### III. NETWORK CAPACITY

For a CDMA network with information rate of $R$ bits/s, spread signal bandwidth of $W$, voice activity factor of $\alpha$, background noise spectral density of $N_0$ and inter-cell interference factor per user $\kappa$, the bit energy to interference density ratio is given by [6]:

$$\frac{E_b}{I_0} = \frac{E_b/N_0}{\alpha(E_b/N_0)(n - 1 + n\kappa)/(W/R) + 1}. \quad (4)$$

To achieve a required bit error rate we must have $\frac{E_b}{I_0} \geq \Gamma$ for some constant $\Gamma$. The above formula assumes that all cells have $n$ calls simultaneously in progress. Let $\kappa_{ji}$ denote the per user inter-cell interference factor of cell $j$ to cell $i$, so that $n_j$ users in cell $j$ produce an amount of interference in cell $i$ equal to $n_j \kappa_{ji}$. The $\kappa_{ji}$ are calculated only once for any given network topology and take into consideration the interference effects from adjacent and non-adjacent cells. Assuming a total of $M$ cells with $n_i$ calls in each cell, we can rewrite equation (4) as:

$$\frac{E_b}{N_0} \left(n_i - 1 + \sum_{j=1}^{M} n_j \kappa_{ji}\right)/(W/R) + 1 \geq \Gamma$$

for $i = 1, ..., M$. \quad (5)

$$n_i + \sum_{j=1}^{M} n_j \kappa_{ji} \leq \frac{W/R}{\alpha} \left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0}\right) + 1 \leq c_{eff}$$

for $i = 1, ..., M$. \quad (6)

The right hand side of equation (6) is a constant, determined by system parameters and by the desired maximum bit error rate, and can be regarded as the total number of effective channels, $c_{eff}$, available to the system.

As can be seen from equation (6), the capacities of the CDMA cells in a network must be considered jointly. The capacity of a cell is the maximum number of users that can be supported simultaneously subject to a required quality of service. Thus the notion of capacity in a CDMA network is that of a capacity region, i.e., the set of all feasible user configurations satisfying equation (6). This capacity region indicates all trade-offs in capacity between the cells in a network.

**Uniform Capacity:** It is usually desirable to make capacity the same in all the cells. Thus small and large cells are used depending on the user distribution density. We define uniform capacity as the requirement that all cells have equal capacity, i.e., $n_i = c_1$ for all $i$.

**Two-Level Capacity:** Sometimes more capacity might be needed in certain cells and less in others. We define a two-level capacity as the requirement that $m_1$ cells have equal capacity $c_1$ and $m_2$ cells have equal capacity $c_2 = k c_1$ where $m_1 + m_2 = M$. 

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IV. POWER COMP. FACTOR

Consider a network with a large cell $j$ adjacent to a small cell $i$ and consider two users $A$ and $B$ both located at the boundary of these two cells as shown in Fig. 2. User $A$ will cause a lot of interference to user $B$. This is because users transmit power proportional to distance raised to the path loss exponent. Thus a small cell adjacent to a large cell will experience a great deal of interference — which causes a significant reduction in the capacity of the small cell.

This problem can be resolved by adjusting the nominal power of the users in every cell by a factor we call the power compensation factor (PCF), in order to make the signal to interference ratio in small cells comparable to the signal to interference ratio in large cells. If cell $j$ has a PCF $\beta_j$, its users' signal power have increased by a factor of $\beta_j$. Thus the new inter-cell interference of cell $j$ to cell $i$ becomes:

$$I_{ji} = E \int \int_{cell_i} \frac{\beta_j r_m^m(x,y)10^{rac{C_r}{10}}}{r_m^m(x,y)/X_i^2} \rho_j dA(x,y).$$  \hspace{1cm} (7)

The new inter-cell interference factor per user becomes $\beta_j \kappa_{ji}$. Thus once the original inter-cell interference factors have been calculated, changing the power compensation factors for the cells does not require recalculation of the original $\kappa_{ji}$ since the interference $I_{ji}$ is linear in $\beta_j$. Equation (6) now becomes:

$$n_i + \sum_{j=1}^{M} n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff}$$  \hspace{1cm} \text{for } i = 1, ..., M. \hspace{1cm} (8)

Calculation of the Implied Cost

Increasing the PCF of a cell increases the signal to interference ratio of that cell thereby increasing the capacity of that cell. On the other hand, it will also increase the interference into its adjacent cells thereby reducing the capacity of those cells. Therefore, we wish to find the optimal values for the PCF’s that will maximize the capacity of the entire network. We will first need to calculate the implied costs, the derivatives of the capacity function with respect to the power compensation factors. They capture the effect of increases in the power compensation factor of one cell on the capacity of the entire network.

For the uniform capacity case, we define $n_i = n_j = c_1$. We can rewrite equation (8) as:

$$c_1 = \min_i \left[ \frac{c_{eff}}{1 + \frac{1}{\sum_{j=1}^{M} \beta_j \kappa_{ji}}} \right]$$  \hspace{1cm} \text{for } i = 1, ..., M. \hspace{1cm} (9)

For numerical purposes, we allow $c_1$ to be a real number. Let $i^*$ denote the index that minimizes equation (9). The derivative of the capacity function, $c_1$, with respect to a power compensation factor, $\beta_{k}$, is:

$$\frac{dc_1}{d\beta_k} = \frac{-c_{eff} \beta_{i^*} \kappa_{k,i^*}}{\left( \beta_{k} + \sum_{j=1}^{M} \beta_j \kappa_{ji} \right)^2} \text{ if } k \neq i^*, \hspace{1cm} (10)$$

$$\frac{dc_1}{d\beta_{k}} = \frac{c_{eff} \sum_{j=1}^{M} \beta_j \kappa_{jk} - c_{eff} \beta_k \kappa_{kk}}{\left( \beta_k + \sum_{j=1}^{M} \beta_j \kappa_{jk} \right)^2} \text{ if } k = i^*. \hspace{1cm} (10)$$

Maximization of Capacity

We formulate a constrained nonlinear optimization problem where the objective function is the maximization of the capacity of the cells in the network. The independent variables are the power compensation factors. The optimization problem for uniform capacity becomes:

$$\max_{\beta_i} \quad M c_1,$$

subject to $1 \leq \bar{\beta} \leq \bar{\beta}^{max}$,

$$n_i + \sum_{j=1}^{M} n_j \frac{\beta_j \kappa_{ji}}{\beta_i} \leq c_{eff},$$  \hspace{1cm} \text{for } i = 1, ..., M. \hspace{1cm} (11)

Each value of $\beta^{max}$ should be chosen so that the required transmit power of the mobiles in cell $i$
Figure 3: Seven cell CDMA network: four small cells and 3 large cells.

does not exceed the maximum power that a mobile can transmit. Thus values of $\beta_{\text{max}}$ are different and inversely proportional to the radius of cell $i$ since the transmit power of the mobiles is directly proportional to the radius of the cells. For two-level capacity, the objective function to maximize is $m_1c_1 + m_2k_c$ over $\beta$ and $\bar{n}$.

The solution for the above optimization problems give the capacity that the network can handle and the optimizing values of the power compensation factors. The optimization is achieved by using the implied costs in a gradient descent algorithm that gives the direction in which to vary the power compensation factors to get the desired maximization [7]. The values of $\kappa_{ij}$ are calculated only once for a given network topology and do not change as we vary the $\beta_i$.

V. RESULTS

The following results have been obtained for the seven cell CDMA network shown in Fig. 3. Our network is made up of 4 small cells (with Forward Pilot Power equal to 1 Watt) and 3 large cells (with Forward Pilot Power equal to 12 Watts). The seven base stations are located at (easting, northing) of (9000, 3250), (12000, 3000), (9500, 6000), (12500, 6500), (4000, 4000), (4500, 9000), and (10500, 10000) meters where the (0, 0) reference is located in the lower left corner of the figure. All seven base stations use omni-directional antennas.

We assume the following for the analysis. We use the COST-231 propagation model [8] with a carrier frequency of 1800 MHz, average base station height of 30 m and average mobile height of 1.5 m. We set the path loss coefficient to 4, the shadow fading standard deviation to 6 dB, with Rayleigh fading on. The processing gain, $\frac{K}{K}$, is 21.1 dB; the bit energy to interference ratio $\frac{E_b}{I_0}$ is 7 dB; the interference to background noise ratio, $\frac{I_0}{N_0}$, is 10 dB. The voice activity factor is 0.375. To implement the integrations given in equation (3), the whole area is divided into small grids of size 150 by 150 m$^2$ and stored in matrix form. The inter-cell interference factors, $\kappa_{ij}$, from cell $i$ to cell $j$ are given in Table 1.

The following results illustrate the maximization of uniform capacity and two-level capacity. Initially all the power compensation factors are set to 1, and the values of $\bar{n}_i$ are calculated. For the optimization, we use $\beta_{\text{max}} = 12$ for small cells and $\beta_{\text{max}} = 1$ for large cells. The optimal values of the power compensation factors, $\beta_i$, for the uniform capacity case are given in Table 2. The optimization of the power compensation factors has accomplished two important goals. The sum of the capacities in all the cells has increased from 77 to 140, and the capacity of each cell has increased from 11 to 20. Results for the two-level capacity model are also presented in Table 2. We required that small cells have capacity $c_1$ and that large cells have capacity $c_2$. We also required that $c_1 = 2c_2$. The results show that, first, the sum of the capacities in all the cells has increased from 77 to 121. But more importantly the capacity in the small cells has increased from 14 to 22, and in the large cells it has increased from 7 to 11.

In Fig. 4 we demonstrate three cases of different capacity allocations. In Case 1, equation (11) is modified by removing the uniform capacity constraint and

<p>| Table 1: The interference $\kappa_{ij}$ of cell $i$ on cell $j$. |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>BS-1</th>
<th>BS-2</th>
<th>BS-3</th>
<th>BS-4</th>
<th>BS-5</th>
<th>BS-6</th>
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<td>0.0623</td>
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<td>0.1213</td>
<td>0.5541</td>
<td>0.0628</td>
<td>0.0038</td>
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<td>$\kappa_{15}$</td>
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<td>0.0784</td>
<td>0.1431</td>
<td>0.5459</td>
<td>0.0925</td>
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<td>$\kappa_{16}$</td>
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<td>0.0327</td>
<td>0.4147</td>
<td>0.0437</td>
<td>0.2353</td>
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<td>$\kappa_{17}$</td>
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<td>0.4646</td>
<td>0.0324</td>
<td>0.3323</td>
<td>0.1510</td>
</tr>
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</table>

<p>| Table 2: Uniform and two-level capacity ($c_1 = 2c_2$), for unoptimized and optimized PCFs. |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>$\beta_{1}$</th>
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<td>$\beta_{5}$</td>
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<td>$\beta_{6}$</td>
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<td>$\beta_{7}$</td>
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VI. CONCLUSION

The effect of reverse power levels on the capacity of CDMA systems was analyzed. A uniform capacity model and a two-level capacity model was presented. We introduced the notion of Power Compensation Factor which enables small cells to overcome the interference from adjacent large cells. We derived the implied costs of the capacity function with respect to the power compensation factors. These implied costs captured the effect of increasing or decreasing the power compensation factor of one cell on the capacity of the entire network. They allow the network administrator to flexibly allocate and distribute capacity between cells. Finally optimal values of the power compensation factors were calculated that maximized the capacity of the entire network.

REFERENCES


