

Cell Placement in a CDMA Network

Robert G. Akl, Manju V. Hegde, Mort Naraghi-Pour*, Paul S. Min

Department of Electrical Engineering,

Washington University, St. Louis, MO 63130.

*Department of Electrical and Computer Engineering,
Louisiana State University, Baton Rouge, LA 70803.

Abstract— Traditional design rules, wherein cells are dimensioned in order to get an equal amount of demand in each cell are not directly applicable to CDMA networks where large cells can cause a lot of interference to adjacent small cells. In order to enable iterative cell placement we use a computationally efficient iterative process to calculate the inter-cell and intra-cell interferences as a function of pilot-signal power and base station location. These techniques enable us to improve the placement of cells in a CDMA network so as to enhance network capacity. We show examples of how networks using this design technique will provide higher capacity than ones designed using conventional techniques.

Keywords— CDMA, Capacity Optimization, Implied Costs.

I. INTRODUCTION

THE capacity of a single cell in a cellular code-division multiple access (CDMA) network depends on the interference of users within that cell (intra-cell interference) as well as on the interference of users in adjacent cells (inter-cell interference). Thus the number of simultaneous calls that can be handled within one cell depends on the number of simultaneous calls in many cells in the network. This interference limitation makes the cell placement design in CDMA networks particularly difficult: the problem of placing cells in a region with a given user profile would require the calculation of the inter-cell interference which depends on the cell geometry and the number of users in the adjacent cells. This problem is not present in networks that use fixed channel assignment algorithms wherein co-channel interference is eliminated by using different frequency sets in adjacent cells thereby separating the problem of cell placement and cell capacity. In such networks, the design rule of thumb is to place cells so that each cell will have a constant demand. Thus, smaller cells are used in areas of high demand while larger cells are used in areas of low demand. This methodology of designing a network topology is not directly applicable to a CDMA network. In a CDMA network, the transmit power of users is proportional to distance (from the base station) raised to the path loss exponent. Consequently, in networks where large cells are adjacent to small cells, users at the boundaries of large cells cause a lot of interference to users in small cells. This causes a significant reduction in the capacity of the small cells. One approach to alleviate this

problem is by increasing the nominal power of the users in the small cells [1] [2]. On the other hand, by controlling the transmitted pilot-signal power and adjusting the location of the base stations, the coverage region of each cell is controlled which in turn controls the intra-cell and inter-cell interference. Given a fixed configuration of user distribution, we try and place a given number of cells in order to maximize capacity. We evaluate the capacity of the entire network as a function of base station location and transmitted pilot-signal power and present an optimization framework which allows us to maximize capacity. We develop design rules that apply to general user configurations. We validate our design rules by presenting comparative capacity results for networks that are designed by our method versus those designed with conventional rules.

II. INTER-CELL INTERFERENCE

Consider two cells i and j . We assume that each user is always communicating with and its power controlled by the base station that has the highest received power at the user. Let C_j denote the region where the received pilot-signal power from base station j is the highest among all base stations. A user located at coordinates (x, y) is at distance $r_j(x, y)$ from base station j . Let n_j be the number of users in cell j and $\text{Area}(C_j)$, the area of cell j . It is assumed that the power control mechanism overcomes both large scale path loss and shadow fading. It does not, however, overcome the fast fluctuations of the signal power associated with Rayleigh fading [3]. The propagation loss of a user in cell j is modeled as the product of the m th power of distance and a log-normal component representing shadowing losses. Now let X_i denote the Rayleigh random variable that represents the fading on the path from this user to cell i . The average of X_i^2 is the log-normal fading on that path, i.e., $E[X_i^2|\zeta_i] = 10^{-\zeta_i/10}$ [4], where ζ_i is the decibel attenuation due to shadowing, and has zero mean and standard deviation σ_s . Consequently, the relative average interference at cell i caused by all users in cell j is given by [5]:

$$I_{ji} = E \left[\int \int_{C_j} \frac{r_j^m(x, y) 10^{\zeta_i/10}}{r_i^m(x, y) / X_i^2} \frac{n_j}{\text{Area}(C_j)} dA(x, y) \right]. \quad (1)$$

Let κ_{ji} denote the per user inter-cell interference factor of cell j to cell i , i.e., $\kappa_{ji} = \frac{I_{ji}}{n_j}$.

This work was partially supported by the McDonnell Foundation and by the National Science Foundation under Grant Number NCR 9706545.

III. CDMA NETWORK CAPACITY

Consider a multi-cell CDMA network with spread signal bandwidth of W , information rate of R bits/s, voice activity factor of α , background noise spectral density of N_0 . Assuming a total of M cells with n_i calls in cell i , the bit energy to interference density ratio in cell i is given by [6]:

$$\left(\frac{E_b}{I_0}\right)_i = \frac{E_b/N_0}{\alpha(E_b/N_0) \left(n_i - 1 + \sum_{j=1}^M n_j \kappa_{ji}\right) / (W/R) + 1} \quad \text{for } i = 1, \dots, M. \quad (2)$$

To achieve a required bit error rate we must have $\left(\frac{E_b}{I_0}\right)_i \geq \Gamma$ for some constant Γ . Thus the number of users in every cell must satisfy:

$$n_i + \sum_{j=1}^M n_j \kappa_{ji} \leq \frac{W/R}{\alpha} \left(\frac{1}{\Gamma} - \frac{1}{E_b/N_0}\right) + 1 \triangleq c_{eff} \quad \text{for } i = 1, \dots, M. \quad (3)$$

A set of users (n_1, \dots, n_M) satisfying the above equation is said to be a feasible user configuration that satisfies the $\frac{E_b}{I_0}$ constraint. The right hand side of (3) is a constant, determined by system parameters and by the desired maximum bit error rate, and can be regarded as the total number of effective channels, c_{eff} , available to the system. As can be seen from (3), the capacities of the CDMA cells in a network must be considered jointly. Thus the notion of capacity in a CDMA network is that of a capacity region. This capacity region indicates all trade-offs in capacity between the cells in a network.

Let n be the total number of users in the network. Let $\rho(x, y)$ be the relative user density at (x, y) . The number of users in cell i is:

$$n_i = n \frac{\int_{C_i} \rho(x, y) dA(x, y)}{\int \bigcup_{j=1}^M C_j \rho(x, y) dA(x, y)}. \quad (4)$$

Combining equations (3) and (4), a feasible user configuration satisfying both the $\frac{E_b}{I_0}$ constraint and the user distribution constraint must satisfy:

$$n\theta_i(C_i) + \sum_{j=1}^M n\theta_j(C_j)\kappa_{ji} \leq c_{eff} \quad \text{for } i = 1, \dots, M. \quad (5)$$

$$\text{where } \theta_i(C_i) = \frac{\int_{C_i} \rho(x, y) dA(x, y)}{\int \bigcup_{j=1}^M C_j \rho(x, y) dA(x, y)}.$$

The capacity of the network becomes:

$$n = \min_i \left[\frac{c_{eff}}{\theta_i(C_i) + \sum_{j=1}^M \theta_j(C_j)\kappa_{ji}} \right] \quad \text{for } i = 1, \dots, M, \quad (6)$$

IV. CELL REGION

Let $L_i = (x_i, y_i)$ be the coordinates of base station i , and T_i , the transmitted pilot-signal power of base station i . Let \mathcal{A}_i denote the set of base stations adjacent to base station i . Given a path loss model, the region of cell i , C_i , is completely determined by $L_i^{(x)}$, $L_i^{(y)}$, $L_j^{(x)}$, $L_j^{(y)}$, T_i , and T_j for $j \in \mathcal{A}_i$.

For example, consider two adjacent cells i and j . Let b_{ij} be the point on the straight line connecting L_i and L_j , where the received pilot power from base station i equals the received pilot power from base station j . Then using the COST-231 model for path loss [7]:

$$b_{ij} = \frac{e^{\gamma(T_i - T_j)}}{1 + e^{\gamma(T_i - T_j)}} d_{ij}, \quad (7)$$

where d_{ij} is the distance between base stations i and j , and γ is a constant which depends on the average base station antenna height and the average mobile antenna height. C_i is the region enclosed in the polygon whose sides pass through b_{ij} and are perpendicular to the line connecting L_i to L_j for $j \in \mathcal{A}_i$ (ignoring edge effects).

A. Implied Cost w.r.t. Transmitted Pilot-Signal Power

In a CDMA network it is important to control the inter-cell interference. Increasing the pilot-signal power of a base station expands the coverage region of that cell thereby increasing the number of users in that cell and thus the intra-cell interference. On the other hand, it will decrease the number of users in the adjacent cells thus decreasing the inter-cell interference on this base station. The opposite effect takes place in the adjacent cells. The inter-cell interference into these cells increases and the intra-cell interference decreases. Therefore, we wish to find the optimal values of the transmitted pilot-signal powers that will maximize the capacity of the entire network. Our approach is to first calculate the implied costs which are the derivatives of the capacity function with respect to T_k 's and use the implied costs in an optimization algorithm. The implied costs capture the effect of increases in the transmitted pilot-signal power of one base station on the capacity of the entire network.

Let i^* denote the index that minimizes (6). For numerical purposes, we allow n to be a real number. The

derivative of the network capacity, n , with respect to θ_k is:

$$\frac{\partial n}{\partial \theta_k} = \begin{cases} \frac{-c_{eff}}{\left(\theta_k + \sum_{j=1}^M \theta_j \kappa_{jk}\right)^2} & \text{if } k = i^* \\ \frac{-\kappa_{ki} \cdot c_{eff}}{\left(\theta_{i^*} + \sum_{j=1}^M \theta_j \kappa_{ji^*}\right)^2} & \text{if } k \neq i^* \end{cases} \quad (8)$$

The derivative of the network capacity, n , with respect to the inter-cell interference factor, κ_{li} , is:

$$\frac{\partial n}{\partial \kappa_{li}} = \begin{cases} \frac{-\theta_l c_{eff}}{\left(\theta_i + \sum_{j=1}^M \theta_j \kappa_{ji}\right)^2} & \text{if } i = i^* \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

From (1):

$$\begin{aligned} \kappa_{li} &= E \left[10^{G_i/10} X_i^2 \right] \frac{1}{\text{Area}(C_l)} \iint_{C_l} f_{li}(x, y) dA(x, y), \\ &= \eta_{li} \frac{1}{\text{Area}(C_l)} F_{li}(C_l, L_i), \end{aligned} \quad (10)$$

where $f_{li}(x, y) = r_l^m(x, y)/r_i^m(x, y)$, $\eta_{li} = E \left[10^{G_i/10} X_i^2 \right]$, and $F_{li}(C_l, L_i) = \iint_{C_l} f_{li}(x, y) dA(x, y)$.

The region of cell l , C_l , is a function of the independent variables L_l, L_p, T_l, T_p where $p \in \mathcal{A}_l$. Thus the partial derivative of κ_{li} with respect to T_k is given by:

$$\frac{\partial \kappa_{li}}{\partial T_k} = \eta_{li} \left[\frac{-1}{\text{Area}^2(C_l)} \frac{\partial \text{Area}(C_l)}{\partial T_k} F_{li}(C_l, L_i) + \frac{1}{\text{Area}(C_l)} \frac{\partial F_{li}(C_l, L_i)}{\partial T_k} \right], \quad (11)$$

if $l = k$ or $l \in \mathcal{A}_k$ and zero otherwise. The inter-cell interference factors $\kappa_{ki}(C_k(T), L_i)$ for $i = 1, \dots, M$, are a function of T_k . Also the inter-cell interference factors $\kappa_{li}(C_l(T), L_i)$ for $l \in \mathcal{A}_k$ and $i = 1, \dots, M$, are a function of T_k . Thus the total derivative of the network capacity, n , with respect to the transmitted pilot-signal power, T_k , is:

$$\begin{aligned} \frac{dn}{dT_k} &= \sum_{i=1}^M \frac{\partial n}{\partial \kappa_{ki}} \frac{\partial \kappa_{ki}}{\partial T_k} \\ &+ \sum_{i=1}^M \sum_{l \in \mathcal{A}_k} \frac{\partial n}{\partial \kappa_{li}} \frac{\partial \kappa_{li}}{\partial T_k} \\ &+ \frac{\partial n}{\partial \theta_k} \frac{\partial \theta_k}{\partial T_k} \\ &+ \sum_{l \in \mathcal{A}_k} \frac{\partial n}{\partial \theta_l} \frac{\partial \theta_l}{\partial T_k}. \end{aligned} \quad (12)$$

B. Implied Cost w.r.t. Base Station Location

Another way we can control the intra-cell and the inter-cell interferences is by controlling the locations of the base stations. Changing the location of a base station changes

the coverage region of that cell and the coverage regions of the adjacent cells. Thus we wish to find the implied costs of the network capacity with respect to base station location and use the implied costs in the optimization algorithm. The implied costs would capture the effect of changing the coordinates of one base station k on the capacity of the entire network.

We start by finding the partial derivative of κ_{li} with respect to $L_k^{(x)}$:

$$\begin{aligned} \frac{\partial \kappa_{li}}{\partial L_k^{(x)}} &= \eta_{li} \left[\frac{-1}{\text{Area}^2(C_l)} \frac{\partial \text{Area}(C_l)}{\partial L_k^{(x)}} F_{li}(C_l, L_i) \right. \\ &\left. + \frac{1}{\text{Area}(C_l)} \frac{\partial F_{li}(C_l, L_i)}{\partial L_k^{(x)}} \right], \end{aligned} \quad (13)$$

if $l = k$ or $l \in \mathcal{A}_k$, and

$$\frac{\partial \kappa_{li}}{\partial L_k^{(x)}} = \eta_{li} \frac{1}{\text{Area}(C_l)} \frac{\partial F_{li}(C_l, L_i)}{\partial L_k^{(x)}}, \quad (14)$$

if $i = k$ and $l \notin \mathcal{A}_k$, and zero otherwise. The inter-cell interference factors $\kappa_{ki}(C_k(L), L_i)$ for $i = 1, \dots, M$, are a function of $L_k^{(x)}$. The inter-cell interference factors $\kappa_{li}(C_l(L), L_i)$ for $l \in \mathcal{A}_k$ and $i = 1, \dots, M$, are a function of $L_k^{(x)}$. Also the inter-cell interference factors $\kappa_{lk}(C_l, L_k)$ for $l = 1, \dots, M$, and $l \notin \mathcal{A}_k$ are a function of $L_k^{(x)}$. Thus the total derivative of the network capacity with respect to the x -coordinate of base station k is:

$$\begin{aligned} \frac{dn}{dL_k^{(x)}} &= \sum_{i=1}^M \frac{\partial n}{\partial \kappa_{ki}} \frac{\partial \kappa_{ki}}{\partial L_k^{(x)}} \\ &+ \sum_{i=1}^M \sum_{l \in \mathcal{A}_k} \frac{\partial n}{\partial \kappa_{li}} \frac{\partial \kappa_{li}}{\partial L_k^{(x)}} \\ &+ \sum_{l=1, l \notin \mathcal{A}_k}^M \frac{\partial n}{\partial \kappa_{lk}} \frac{\partial \kappa_{lk}}{\partial L_k^{(x)}} \\ &+ \frac{\partial n}{\partial \theta_k} \frac{\partial \theta_k}{\partial L_k^{(x)}} \\ &+ \sum_{l \in \mathcal{A}_k} \frac{\partial n}{\partial \theta_l} \frac{\partial \theta_l}{\partial L_k^{(x)}}. \end{aligned} \quad (15)$$

We have a similar expression for $L_k^{(y)}$.

V. MAXIMIZATION OF CAPACITY

We form a constrained nonlinear optimization problem where the objective function is the network capacity. The independent variables are the transmitted pilot-signal powers. Thus the optimization problem is:

$$\begin{aligned} \max_{\mathbf{T}} \quad & n, \\ \text{subject to} \quad & n\theta_i(C_i) + \sum_{j=1}^M n\theta_j(C_j) \kappa_{ji}(C_j, L_i) \leq c_{eff}, \\ & \text{for } i = 1, \dots, M. \end{aligned} \quad (16)$$

The solution for the above optimization problem gives the capacity that the network can handle and the optimizing values of the transmitted pilot-signal powers. The optimization is achieved by using the implied costs in a gradient descent algorithm that gives the direction in which to vary the pilot-signal powers to get the desired maximization.

Another optimization that can be done is for the case when the independent variables are the base station locations. Finally, we maximize the network capacity by optimizing simultaneously the transmitted pilot-signal powers and the base station locations.

The simultaneous optimization problem is:

$$\begin{aligned} \max_{T,L} \quad & n, \\ \text{subject to} \quad & n\theta_i(C_i) + \sum_{j=1}^M n\theta_j(C_j)\kappa_{ji}(C_j, L_i) \leq c_{eff}, \\ & \text{for } i = 1, \dots, M. \end{aligned} \quad (17)$$

VI. RESULTS

The following results have been obtained for the thirty-six cell CDMA network shown in Fig. 1. We assume the following for the analysis. We use the COST-231 propagation model with a carrier frequency of 1800 MHz, average base station height of 30 meters and average mobile height of 1.5 meters. We set the path loss coefficient, m , to 4, the shadow fading standard deviation, σ_s , to 6 dB. The processing gain, $\frac{W}{R}$, is 21.1 dB; the bit energy to interference ratio $\frac{E_b}{I_0}$ is 7 dB; the interference to background noise ratio, $\frac{I_p}{N_0}$, is 10 dB. The voice activity factor, α , is 0.375. The whole area is divided into small grids of size 300 by 300 m^2 . Base station 1 is located at (1000,1000) meters, base station 2 is 3000 meters east of base station 1, base station 3 is 3000 meters east of base station 2, and so forth where the (0, 0) reference is located in the lower left corner of Fig. 1. Base station 7 is located 3000 meters north of base station 1, base station 8 is located 3000 meters north of base station 2, and so forth. The pilot-signal power of all base stations is 1 Watt. Every grid point in the network has a relative user density of 1 per grid point, i.e., a uniform user distribution.

The capacity of this network calculated from (6) is 681 simultaneous users. After running the optimization for transmitted pilot-signal power only, base station location only, and both simultaneously, the pilot-powers, the locations of the base stations, and the network capacity remained unchanged in all three cases. Thus, as expected, for a uniform user distribution, a uniform network layout with equal pilot-signal power, equal distance between adjacent base stations, and equal coverage region is optimal.

We then considered two hot spots as shown in Fig 2. The first hot spot is rectangular in shape and consists of 300 grid points with a relative user density of 3 per grid point. The lower left corner is at (2950, 2950) meters and the upper right corner is at (8650,7150) meters. The second hot spot is circular in shape and consists of 377 grid points

with a relative user density of 2 per grid point. The center is located at (11650, 10450) meters and has a radius of 3000 meters. The rest of the network has a relative user density of 1 per grid point.

The capacity of this network with equal pilot powers is 361 simultaneous users. Base station 9 has the highest amount of intra-cell and inter-cell interference. One would expect to decrease the pilot-signal power of base station 9 in order to reduce the coverage region and thus reduce the intra-cell interference. But the optimization of the pilot-signal powers reveals otherwise as shown in Fig. 3. The optimization requires the pilot power of base station 9 to increase from 1 Watt to 1.617 Watts. Also base station 8 should increase its pilot power from 1 to 1.445 Watts and base station 29 from 1 to 1.5 Watts. This increase in pilot power and thus coverage region of base station 9 does increase the intra-cell interference further. But the inter-cell interference now decreases more giving us a reduction in total interference. The total capacity of the network increases to 372 simultaneous users.

The maximization of network capacity with respect to base station location produces the network shown in Fig. 4. The optimization moves base stations 8, 9, 14, and 15 closer and places them uniformly in the hot spot. A similar relocation takes place for base stations 22, 23, 28, and 29. This optimization increases the network capacity, n , to 385 simultaneous users.

Finally we maximize network capacity with respect to pilot-signal powers and base station locations. The result of this optimization is shown in Fig. 5. The pilot powers of base stations 8, 14, 28, and 29 are increased to 1.1, 1.574, 1.625, and 1.573 Watts respectively. This optimization increases the network capacity to 405 simultaneous users.

VII. CONCLUSION

We show how to maximize the number of simultaneous users in a network by varying the pilot-signal powers and

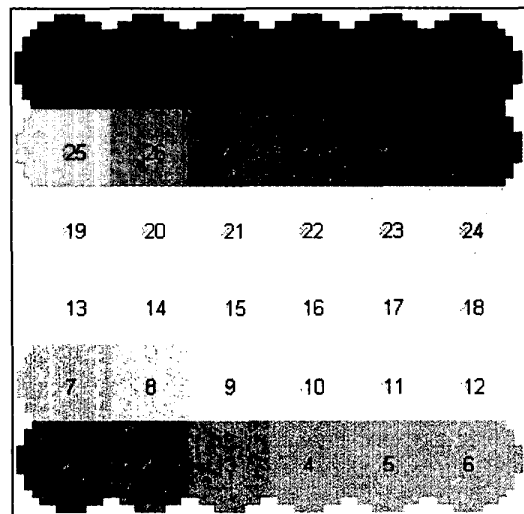


Fig. 1. Thirty-six cell CDMA network with uniform user distribution.

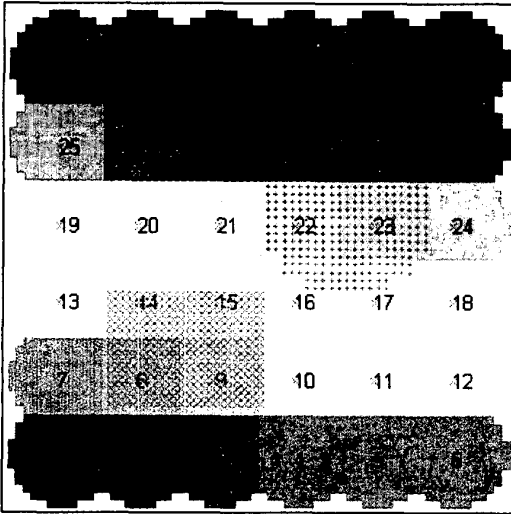


Fig. 2. Thirty-six cell CDMA network with two hot spots.

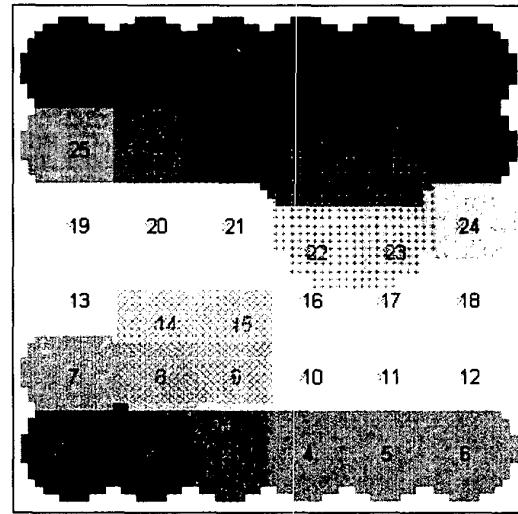


Fig. 4. Thirty-six cell CDMA network with optimized base station location.

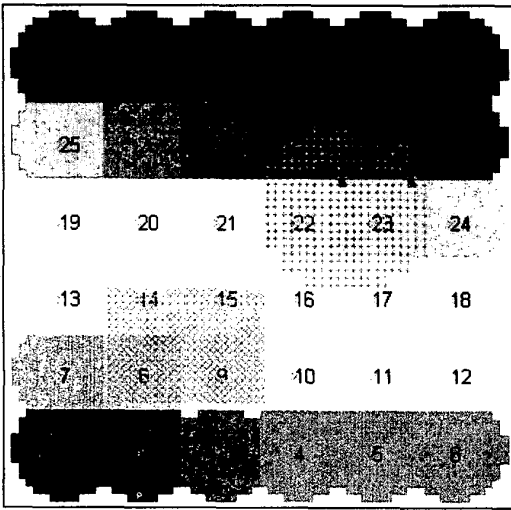


Fig. 3. Thirty-six cell CDMA network with optimized pilot-signal powers.

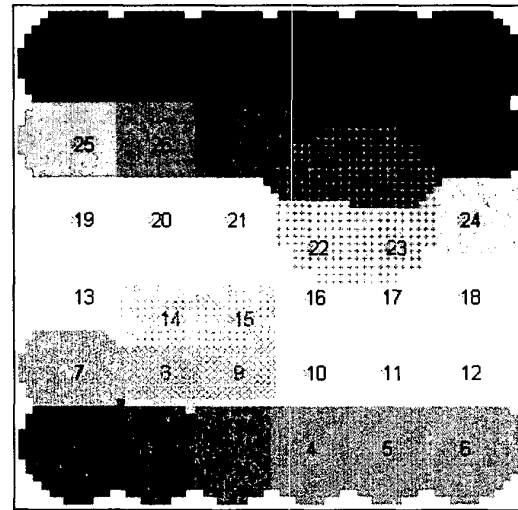


Fig. 5. Thirty-six cell CDMA network with optimized pilot powers and base station location.

the base station locations. We calculate the implied costs of network capacity with respect to pilot-signal powers and base station locations. Implied costs capture the effect of increases in the pilot-signal power of one base station or the relocation of one base station on the capacity of the entire network. They are a tool that allow us to maximize capacity. The results confirm that for a uniform user distribution, a uniform network layout with equal-sized cells is optimal. For a non-uniform distribution, the cells need to be relocated inside the hot spot. The results show that increasing the pilot power of a congested cell is better than decreasing it. Even though the intra-cell interference increases, a greater reduction in inter-cell interference is achieved which yields an increase in overall capacity.

REFERENCES

- [1] R. G. Akl, M. V. Hegde, M. Naraghi-Pour, and P. S. Min, "Flexible Allocation of Capacity in Multi-Cell CDMA Networks," *IEEE Vehicular Technology Conference*, May 16-20, 1999.
- [2] K. Takeo and S. Sato, "Evaluation of a CDMA Cell Design Algorithm Considering Non-uniformity of Traffic and B.S. Locations," *IEICE Trans. Fund.*, Vol E81-A, No 7, pp. 1367-1377, July 1998.
- [3] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, Jr., and C. E. Wheatley, III, "On the capacity of a cellular CDMA system", *IEEE Trans. Veh. Technol.*, pp. 303-312, May 1991.
- [4] H. Susuki, "A Statistical Model for Urban Radio Propagation," *IEEE Trans. Comm.*, Vol 25, pp. 673-680, July 1977.
- [5] R. G. Akl, M. V. Hegde, A. Chandra, and P. S. Min, "CDMA Capacity Allocation and Pricing," Design Document, Washington University, St. Louis Missouri, April 1998.
- [6] David Everitt, "Analytic traffic models of CDMA cellular networks", *ITC 14*, pp. 349-356, 1994.
- [7] Theodore S. Rappaport, *Wireless Communications Principles and Practice*. Prentice Hall, 1996, pp. 116-120.