Approximating User Distributions in WCDMA Networks Using 2-D Gaussian

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ABSTRACT

In this paper, we present an analytical model for approximating the user distributions in multi-cell third generation WCDMA networks using 2-dimensional Gaussian distributions by determining the means and the standard deviations of the distributions for every cell. This allows us to calculate the inter-cell interference and the reverse-link capacity of the network. We compare our model with simulation results and show that it is fast and accurate enough to be used efficiently in the planning process of large WCDMA networks.

Keywords: Inter-cell interference, Capacity, WCDMA, User distribution, 2-D Gaussian distribution.

1. INTRODUCTION

Wideband Code Division Multiple Access (WCDMA) is an air interface that is proposed for third generation wireless networks that provides a vast range of data services with bit rates of up to 2Mbps with varying quality of service requirements. Since CDMA was first introduced in 1989 by QUALCOMM, the number of subscribers has grown to more than 240 million globally. The ability to offer greater capacity, multi-rate transmission with backward compatibility, effortless integration, and easy migrating path to 3G cellular systems, has created the widespread deployment of CDMA systems all over the world [1].

It has been shown in [2], [3], [4], [5] that the capacity of a CDMA network is reverse link limited, and therefore our study is focused on reverse link capacity. One of the principal characteristics of a WCDMA network is that the capacity of the system is a function of the total interference experienced by the network, and is upper bounded by the cell experiencing the most interference. Thus, it is imminent to characterize the total inter-cell interference seen by a single cell in terms of the user distribution in all other cell for determining the capacity in that single cell. Traditionally, the total interference contributed by a cell has been viewed as an approximation, determined by simply multiplying the number of users in that cell by the average interference offered by that cell [6]. In other words, a user placed anywhere within a cell generated the same amount of interference. Clearly, a more realistic approach will use per-user interference as a function of its actual distance to the point of interest. There is a dearth of literature where actual distance was used in the interference model. In [7], even though interference was calculated using actual distance, the capacity calculations were done using mean value of interference. User positions were varied over time, but the number of users was kept constant.

In this paper, we present an analytical model for the approximation of the user distribution in multi-cell WCDMA networks using 2-dimensional Gaussian distributions by determining the means and the standard deviations of the distributions for every cell. Once the user distributions are approximated, the average inter-cell interferences can be determined similar to what was done in [8]. We compare our model with simulation results presented in [9] and show that it is fast and accurate enough to be used efficiently in the planning process of large WCDMA networks.

The remainder of this paper is organized as follows. In section 2, we use the 2-D Gaussian function for modeling user distributions and calculating the average inter-cell interference. In section 3, we compute the capacity of a WCDMA network. Numerical results are presented in section 4. Finally, our conclusions are given in section 5.

2. AVERAGE INTER-CELL INTERFERENCE MODEL USING 2-D GAUSSIAN DISTRIBUTION

It is assumed that each user is always communicating and is power controlled by the base station (BS) that has the highest received power at the user. Let $r_i(x,y)$ and $r_j(x,y)$ be the distance from a user to BS i and BS j, respectively. This user is power controlled by BS j in the cell or region C_j with area A_j, which BS j services. It is assumed that both large scale path loss and shadow fading are compensated by the perfect power control mechanism. Let $I_{ji,t}$ be the average inter-cell interference that all users $n_{j,t}$ using services t with activity factor v_t and received signal S_t at BS j impose on BS i. Modifying the average inter-cell interference given by [8], we have

$$I_{ji,t} = S_t v_t n_{j,t} \, \frac{e^{(\gamma\sigma_s)^2}}{A_j} \iint_{C_j} \frac{r_j^m(x,y)}{r_i^m(x,y)} \, w(x,y) \, dA(x,y), \tag{1}$$

where $\gamma = \ln(10)/10$, σ_s is the standard deviation of the attenuation for the shadow fading, m is the path loss exponent, and $w(x,y)$ is the user distribution density at (x,y).

We define $\kappa_{ji,t}$ to be the per-user (with service t) relative inter-cell interference factor from cell j to BS i, i.e.,

$$\kappa_{ji,t} = \frac{e^{(\gamma\sigma_s)^2}}{A_j} \int\int_{C_j} \frac{r_j^m(x,y)}{r_i^m(x,y)}\ w(x,y)\ dA(x,y). \quad (2)$$

The inter-cell interference density I_{ji}^{inter} from cell j to BS i from all services T becomes

$$I_{ji}^{inter} = \frac{1}{W}\sum_{t=1}^{T}\ I_{ji,t}, \quad (3)$$

where W is the bandwidth of the system. Eq. (3) can be rewritten as

$$I_{ji}^{inter} = \frac{1}{W}\sum_{t=1}^{T}\ S_t v_t\ n_{j,t}\ \kappa_{ji,t}. \quad (4)$$

Thus, the total inter-cell interference density I_i^{inter} from all other cells to BS i is

$$I_i^{inter} = \frac{1}{W}\sum_{j=1,j\neq i}^{M}\ \sum_{t=1}^{T}\ S_t v_t\ n_{j,t}\ \kappa_{ji,t}, \quad (5)$$

where M is the total number of cells in the network.

If the user distribution density can be approximated, then, $\kappa_{ji,t}$ needs to be calculated only once. We model the user distribution by a 2-dimensional Gaussian distribution as follows

$$w(x,y) = \frac{\eta}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2}(\frac{x-\mu_1}{\sigma_1})^2} e^{-\frac{1}{2}(\frac{x-\mu_2}{\sigma_2})^2}, \quad (6)$$

where η is a user density normalizing parameter. We show that by specifying the means μ_1 and μ_2 and the variances σ_1 and σ_2 of the distribution for every cell, we can approximate a wide range of user distributions ranging from uniform to hot spot clusters. We compare these results with simulations and determine the value of η experimentally.

3. WCDMA CAPACITY

In WCDMA, the energy per bit to total interference density at BS i for a service t is given by [10]

$$(\frac{E_b}{I_0})_{i,t} = \frac{\frac{S_t}{R_t}}{N_0\ +\ I_i^{inter}\ +\ I_i^{own}\ -\ S_t v_t}, \quad (7)$$

where N_0 is the thermal noise density, R_t is the bit rate for service t. I_i^{own} is the total intra-cell interference density caused by all users in cell i. Thus I_i^{own} is given by

$$I_i^{own} = \frac{1}{W}\sum_{t=1}^{T}\ S_t v_t n_{i,t}. \quad (8)$$

Let τ_t be the minimum signal-to-noise ratio, which must received at a BS to decode the signal of a user with service t, and S_t^* be the maximum signal power, which the user can transmit. Substituting (5) and (8) into (7), we have for every cell i in the WCDMA network, the number of users $n_{i,t}$ in BS i for a given service t needs to meet the following inequality constraint

$$\tau_t \leq \frac{\frac{S_t^*}{R_t}}{N_0 + \frac{S_t^*}{W}\left[X(i,t)\right]}, \quad (9)$$

where

$$X(i,t) = \sum_{t=1}^{T}\ n_{i,t}v_t + \sum_{j=1,j\neq i}^{M}\sum_{t=1}^{T} n_{j,t}v_t\kappa_{ji,t} - v_t. \quad (10)$$

After rearranging terms, (9) can be rewritten as

$$\sum_{t=1}^{T}\ n_{i,t}v_t + \sum_{j=1,j\neq i}^{M}\sum_{t=1}^{T} n_{j,t}v_t\kappa_{ji,t} - v_t \leq c_{eff}^{(t)}, \quad (11)$$

where

$$c_{eff}^{(t)} = \frac{W}{R_t}\left[\frac{1}{\tau_t} - \frac{R_t}{S_t^*/N_0}\right]. \quad (12)$$

The capacity in a WCDMA network is defined as the maximum number of simultaneous users $(n_{1,t}, n_{2,t}, ..., n_{M,t})$ for all services $t = 1, ..., T$ that satisfy (11).

4. NUMERICAL ANALYSIS

The results shown are for a twenty-seven cell network topology used in [9]. The COST-231 propagation model with a carrier frequency of 1800 MHz, average base station height of 30 meters and average mobile height of 1.5 meters is used to determine the coverage region. The path loss coefficient m is 4. The shadow fading standard deviation σ_s is 6 dB. We assume only one service, i.e., $T = 1$. The processing gain $\frac{W}{R}$ is 21.1 dB. The bit energy to interference ratio threshold, τ, is 9.2 dB. The activity factor, v, is 0.375. These parameters yield c_{eff} of 38.25.

The simulator used for comparison is an extension of the software tools CDMA Capacity Allocation and Planning (CCAP) [11]. CCAP, written in MATLAB, was developed at Washington University in St. Louis for numerical analysis of optimization techniques developed in [8] to compute the capacity of CDMA networks. We extended CCAP for WCDMA networks and used the 2-dimensional Gaussian function for $w(x,y)$. In what follows, we show that by using 2-D Gaussian distribution, we can model many different scenarios including users uniformly distributed, users clustered at the center of the cells, and users at the cells' boundaries. We verified the results in [9], where actual distances were used to simulate real-time users entering the network for the calculation of interference.

We analyzed the network with different values of σ_1 and σ_2, while keeping μ_1 and μ_2 equal to zero in (6). Table I shows the maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 5000 to 15000 while μ_1=0 and μ_2=0. This results in users spread out (almost uniformly) in the cells. Fig. 1 shows the 2-D Gaussian approximation of users uniformly distributed in the cells with σ_1=σ_2=12000. The total number of users is 548. This compares well with simulation results presented in Fig. 2, which yields a total number of users equal to 554 when they are placed uniformly in the cells.

Table II shows the maximum number of users in every cell for the 27 cell WCDMA network as the values of σ_1 and σ_2 are increased from 100 to 400 while μ_1=0 and μ_2=0. This results in users densely clustered around the

TABLE I

THE MAXIMUM NUMBER OF USERS IN EVERY CELL FOR THE 27 CELL WCDMA NETWORK AS THE VALUES OF σ_1 AND σ_2 ARE INCREASED FROM 5000 TO 15000 WHILE μ_1=0 AND μ_2=0. THIS RESULTS IN USERS SPREAD OUT (ALMOST UNIFORMLY) IN THE CELLS.

σ=σ_1, σ_2	5000	7000	10000	12000	15000	Uni Dist
$Cell_1$	18	18	18	18	18	18
$Cell_2$	18	18	18	18	18	18
$Cell_3$	18	18	18	17	17	17
$Cell_4$	18	18	18	17	17	17
$Cell_5$	18	18	18	18	18	18
$Cell_6$	18	18	18	17	17	17
$Cell_7$	18	18	18	17	17	17
$Cell_8$	18	18	18	18	18	18
$Cell_9$	18	17	17	17	17	17
$Cell_{10}$	22	21	21	21	21	21
$Cell_{11}$	22	22	22	21	21	21
$Cell_{12}$	22	21	21	21	21	21
$Cell_{13}$	17	17	17	17	17	17
$Cell_{14}$	18	18	18	18	18	18
$Cell_{15}$	18	17	17	17	17	17
$Cell_{16}$	22	21	21	21	21	21
$Cell_{17}$	22	22	21	21	21	21
$Cell_{18}$	22	21	21	21	21	21
$Cell_{19}$	18	17	17	17	17	17
$Cell_{20}$	25	25	25	25	25	25
$Cell_{21}$	25	25	24	24	24	24
$Cell_{22}$	25	25	24	24	24	24
$Cell_{23}$	25	25	25	25	25	25
$Cell_{24}$	25	25	25	25	25	25
$Cell_{25}$	25	25	24	24	24	24
$Cell_{26}$	25	25	24	24	24	24
$Cell_{27}$	25	25	25	25	25	25
Total Users	565	558	553	548	548	548

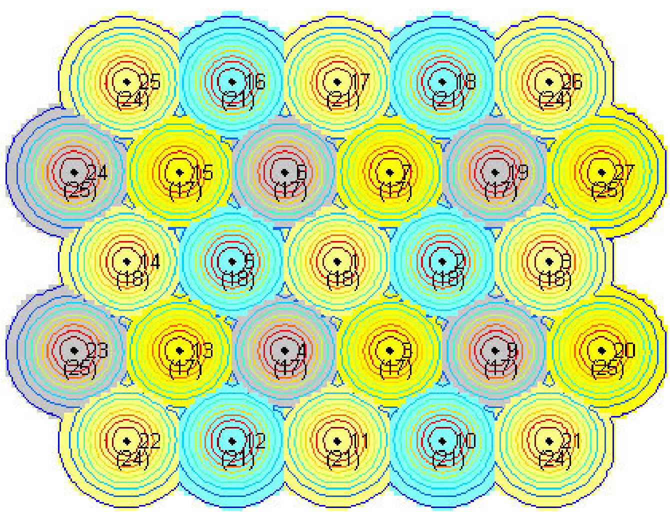

Fig. 1. 2-D Gaussian approximation of users uniformly distributed in the cells. σ_1=σ_2=12000, μ_1=μ_2=0. The maximum number of users is 548.

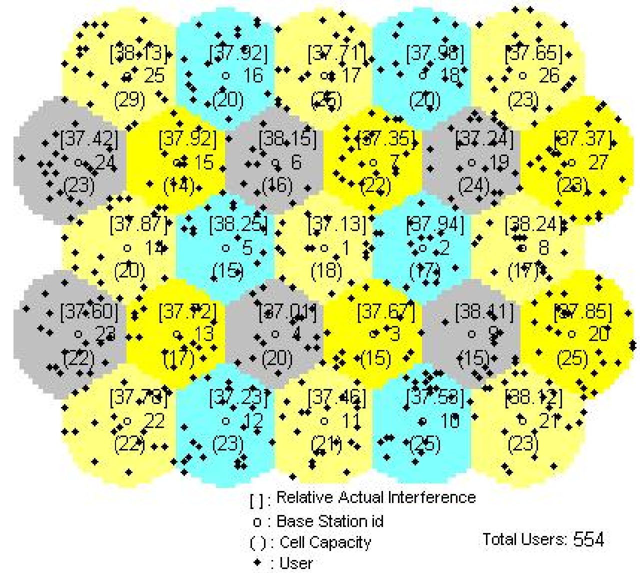

Fig. 2. Simulated network capacity where users are uniformly distributed in the cells. The maximum number of users is 554.

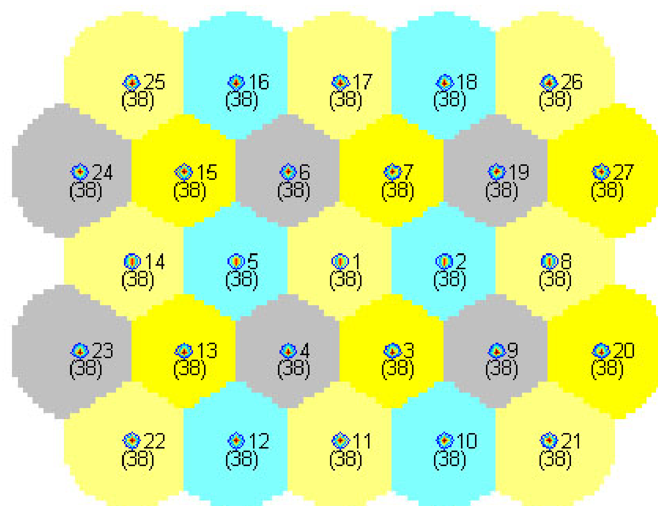

Fig. 3. 2-D Gaussian approximation of users densely clustered around the BSs. σ_1=σ_2=100, μ_1=μ_2=0. The maximum number of users is 1026.

BSs. Fig. 3 shows the 2-D Gaussian approximation with σ_1=σ_2=100. The maximum number of users is 1026. This compares exactly with simulation results presented in Fig. 4, which yields a total number of users equal also to 1026. In this configuration, the users cause the least amount of interference to the network, by reducing the power gain required to maintain a desired signal-to-noise ratio.

Fig. 5 shows the 2-D Gaussian approximation of users clustered at the boundaries of the cells. The values of σ_1, σ_2, μ_1, and μ_2 may be different in the different cells and are given in Table III. The maximum number of users is 133. This result is close to what was attained through simulation. The maximum network capacity was made low by having the simulator place the users such that they cause maximum interference to the network. The simulation yielded a total capacity of 108 users, with only 4 users in each cell. The

TABLE II

THE MAXIMUM NUMBER OF USERS IN EVERY CELL FOR THE 27 CELL WCDMA NETWORK AS THE VALUES OF σ_1 AND σ_2 ARE INCREASED FROM 100 TO 400 WHILE μ_1=0 AND μ_2=0. THIS RESULTS IN USERS DENSELY CLUSTERS AROUND THE BSS.

σ=σ_1, σ_2	$\sigma = 100$	$\sigma = 200$	$\sigma = 300$	$\sigma = 400$
$Cell_1$	38	38	37	34
$Cell_2$	38	38	37	34
$Cell_3$	38	38	37	35
$Cell_4$	38	38	37	35
$Cell_5$	38	38	37	34
$Cell_6$	38	38	37	35
$Cell_7$	38	38	37	35
$Cell_8$	38	38	37	35
$Cell_9$	38	38	37	35
$Cell_{10}$	38	38	37	36
$Cell_{11}$	38	38	37	36
$Cell_{12}$	38	38	37	36
$Cell_{13}$	38	38	37	35
$Cell_{14}$	38	38	37	35
$Cell_{15}$	38	38	37	35
$Cell_{16}$	38	38	37	35
$Cell_{17}$	38	38	37	35
$Cell_{18}$	38	38	37	35
$Cell_{19}$	38	38	37	35
$Cell_{20}$	38	38	37	36
$Cell_{21}$	38	38	38	36
$Cell_{22}$	38	38	38	37
$Cell_{23}$	38	38	38	36
$Cell_{24}$	38	38	38	36
$Cell_{25}$	38	38	37	36
$Cell_{26}$	38	38	37	36
$Cell_{27}$	38	38	37	36
Total Users	1026	1026	1003	954

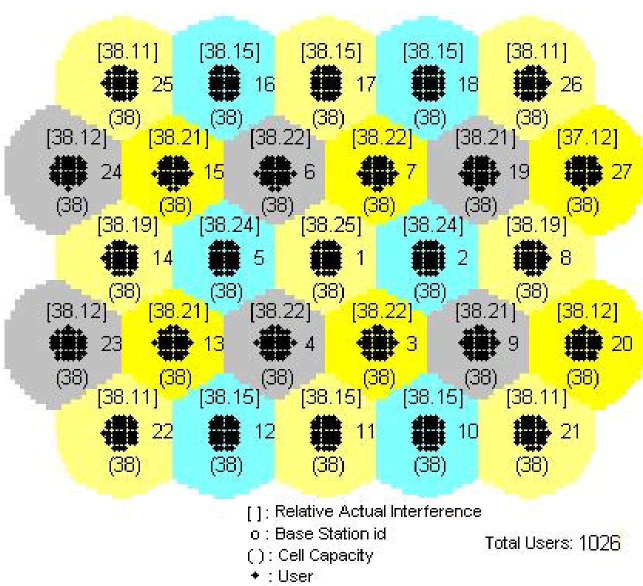

Fig. 4. Simulated network capacity where users are densely clustered around the BSs causing the least amount of inter-cell interference. The maximum number of users is 1026 in the network.

TABLE III

THE VALUES OF σ_1, σ_2, μ_1, AND μ_2 FOR THE 2-D GAUSSIAN APPROXIMATION OF USERS CLUSTERED AT THE BOUNDARIES OF THE CELLS AS SHOWN IN FIG. 5. THE MAXIMUM NUMBER OF USERS IS 133.

	μ_1	σ_1	μ_2	σ_2
$Cell_1$	-1400	300	-900	300
$Cell_2$	-1400	300	800	300
$Cell_3$	-1400	300	800	300
$Cell_4$	0	300	-1700	300
$Cell_5$	0	300	-1600	300
$Cell_6$	1300	300	-800	300
$Cell_7$	-1400	300	900	300
$Cell_8$	-1300	300	900	300
$Cell_9$	0	300	1500	300
$Cell_{10}$	0	300	1600	300
$Cell_{11}$	0	300	1550	300
$Cell_{12}$	-1400	300	900	300
$Cell_{13}$	0	300	1500	300
$Cell_{14}$	1300	300	900	300
$Cell_{15}$	1300	300	-800	300
$Cell_{16}$	-1350	300	-850	300
$Cell_{17}$	-1400	300	-900	300
$Cell_{18}$	0	300	-1600	300
$Cell_{19}$	-1400	300	-800	300
$Cell_{20}$	-1400	300	-800	300
$Cell_{21}$	-1350	300	800	300
$Cell_{22}$	0	300	1600	300
$Cell_{23}$	1350	300	800	300
$Cell_{24}$	1400	300	-800	300
$Cell_{25}$	0	300	-1700	300
$Cell_{26}$	0	300	-1600	300
$Cell_{27}$	-1350	300	-850	300

pattern seen in Fig. 6 shows that the simulator placed the users at the extreme corners of their respective cells. The placement at extremities would require users to increase their power gain causing a lot more interference to other users.

5. CONCLUSIONS

We presented an analytical model for approximating the user distributions in multi-cell WCDMA networks using 2-dimensional Gaussian distributions by determining the means and the standard deviations of the distributions for every cell. This allowed for the calculation of the inter-cell interference and the reverse-link capacity of the network. We compared our model with simulation results and showed that it is fast and accurate enough to be used efficiently in the planning process of large WCDMA networks.

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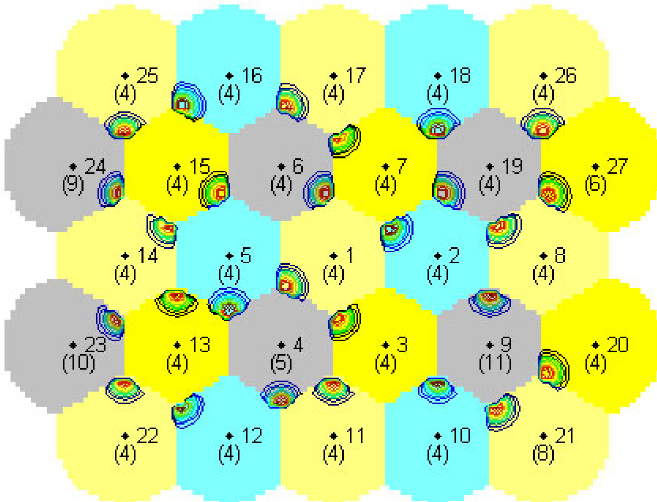

Fig. 5. 2-D Gaussian approximation of users clustered at the boundaries of the cells. The values of σ_1, σ_2, μ_1, and μ_2 may be different in the different cells and are given in Table III. The maximum number of users is 133.

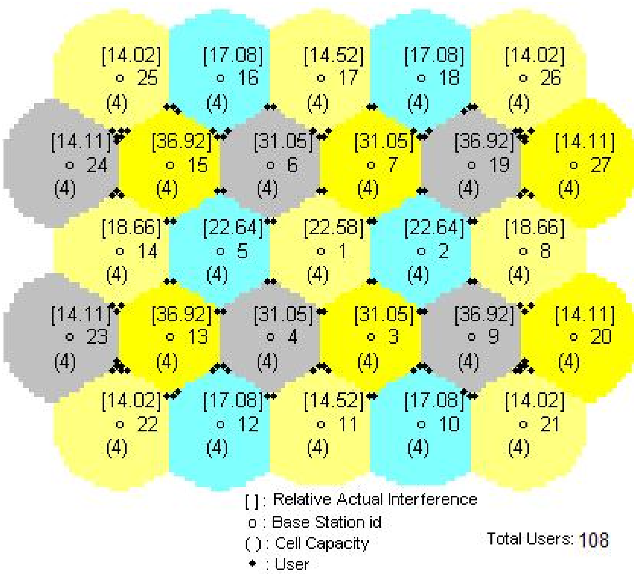

Fig. 6. Simulated network capacity where users are clustered at the boundaries of the cells causing the most amount of inter-cell interference. The maximum number of users is only 108 in the network.

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