Spectrally Efficient Modulation and Turbo Coding for Communication Systems

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Presentation Outline:

• The digital communication problem.
• Limits on communication.
• Modulation.
• Turbo coding.
• Spectral efficient modulation with TC.
• Parity Assisted Decoding.
• Summary.
The communication problem

• Transmission of *information* from one point to another.
• Transmission can be from one place to another,
• it can also be from one time to another (i.e. storage).
Limited resources

• In communication, the two scarce resources are bandwidth and power.
• There is huge demand on the usable spectrum.
• Government and international agencies allocate and auction spectrum.
• Battery and power supplies output (as well as our health) dictate available power.

• Extreme cases:
  • Power limited: deep space probes have (almost) infinite BW.
  • BW limited: antennas on cell towers, are assigned very limited BW.
Communication obstacles

- Signal attenuation.
- Receiver noise.
- Channel filtering.
- Multipath fading.
- Interference.
A Communication System

```
sink
    source
    encoder
    chan. encoder
    modulator
    channel
    demodulator
    chan. decoder
    source decoder
    sink
```
The channel encoder

- It expands the data by adding redundancy.
- Redundancy is added in a systematic fashion to correct errors introduced by the channel upstream.
- Improves in BER performance at the expense of increased bandwidth.
- The two major types of channel codes are:
  - block codes, ex. Hamming, RS and BCH, and
  - convolutional codes.
QAM modulation

- In QAM modulation, two quadrature sinusoidal waveforms are changed simultaneously.
- A transmitted waveform can be of the form
  \[ s(t) = A_n g(t) \cos 2\pi f_c t - B_n g(t) \sin 2\pi f_c t. \]
- \( A_n \) and \( B_n \) are varied according to the transmitted bits.
QAM modulation

Example of QAM modulation: QAM-16
QAM modulation

Decision boundaries are vertical and horizontal lines.

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Dissertation Defense – p. 10
QAM modulation:

Example of received data in QAM-16 square.
QAM modulation

QAM: Hexagonal constellation. Average $E_b$ is lower than that of square QAM. Decision regions are no longer straight lines.
Shannon’s Limit

The capacity of a AWGN channel of bandwidth $W$ and signal to noise ratio of $E_s/N_0$ is

\[ C = W \log_2(1 + E_s/N_0) \]  
bits per second.
Shannon’s Limit ...

\[ C = W \log_2 (1 + \frac{E_s}{N_0}). \]

- Rate, \( R \), must be less than or equal to \( C \) for reliable communication.
- Let \( \eta \overset{\text{def}}{=} \frac{R}{W} \) then

\[ \frac{E_b}{N_0} > \frac{(2^\eta - 1)}{\eta}, \]

- where \( E_s = \eta E_b \).
Shannon’s Limit
Operating points for some modulation schemes

Operating points for BER of $10^{-5}$. 
Turbo encoder

\[ x(n) \]

- Interleaver
- RSC
- RSC
Recursive Systematic Convolutional (RSC) encoder

\[ x(n) \]

\[ y_1(n) \]
Interleaver example

\[
x_0 \\
x_1 \\
x_2 \\
\vdots \\
x_{n-1}
\]

\[
y_0 \\
y_1 \\
y_2 \\
\vdots \\
y_{n-1}
\]
**Turbo codes: interleavers**

Input $\mathbf{x} = [x_0, x_1, \ldots, x_{15}]$, can be put in a $4 \times 4$ array as

\[
\begin{array}{cccc}
    x_0 & x_1 & x_2 & x_3 \\
    x_4 & x_5 & x_6 & x_7 \\
    x_8 & x_9 & x_{10} & x_{11} \\
    x_{12} & x_{13} & x_{14} & x_{15} \\
\end{array}
\]

then read column-wise as

$\mathbf{y} = [x_0, x_4, x_8, x_{12}, x_1, \ldots, x_{15}]$. 
Turbo codes: interleavers ...

\[
\begin{align*}
  x_0 & \rightarrow x_1 & x_2 & \rightarrow x_3 \\
  x_4 & \quad x_5 & x_6 & \quad x_7 \\
  x_8 & \quad x_9 & x_{10} & \quad x_{11} \\
  x_{12} & \rightarrow x_{13} & x_{14} & \rightarrow x_{15}.
\end{align*}
\]
Turbo codes: interleavers

- For large blocks, random interleavers perform well.
- Short interleavers require careful design.
- Interleaver design is an active area of research.
- No acceptable measure of “goodness”. Interleavers are evaluated via simulation.
- Mathematically, an interleaver is a permutation matrix $P$. The inverse of $P$ is $P^T$. 
Combined High Spectral Efficiency modulation and Turbo coding

- Group every $k$ bits from the systematic output.
- Map the bits into a symbol in $M = 2^k$ point QAM constellation.
- The points are mapped according to Gray coding.
- Find corresponding waveform and transmit.
- The same procedure is repeated for the outputs of the RSC’s.
**Turbo codes and QAM**

- Decoding can be done by hard decision or semi-soft decision decoding.
- In hard decision decoding, each received symbol is decoded to the closest point in the constellation.
- (Quantized) bit symbols are extracted from the decoded hard decision symbol.
- Soft information are lost.
- The bit symbols are accumulated and sent to to a MAP decoder.
Turbo codes and QAM: Semi-soft decoding

- In soft decision decoding, the soft information are approximated by the location of the received symbol.
- Functions are introduced to decouple the received symbol into bit symbols.
- Bits in even and odd positions are corrupted with uncorrelated noise.
- The approximation is accurate for high SNR’s.
## Approximating soft information

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b_3b_2b_1b_0
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Approximating soft information

Soft function $r_1(r)$ corresponding to bit $b_1$. 
Approximating soft information

Soft function $r_3(r)$ corresponding to bit $b_3$. 

![Graph showing the soft function $r_3(r)$ corresponding to bit $b_3$.]
Gray coding for QAM 64

\[
\begin{array}{cccccccc}
100011 & 100001 & 101001 & 101011 & 001011 & 001001 & 000001 & 000011 \\
100010 & 100000 & 101000 & 101010 & 001010 & 001000 & 000000 & 000010 \\
100110 & 100100 & 101100 & 101110 & 001110 & 001100 & 000100 & 000110 \\
100111 & 100101 & 101101 & 101111 & 001111 & 001101 & 000101 & 000111 \\
110111 & 110101 & 111101 & 111111 & 011111 & 011101 & 010101 & 010111 \\
110110 & 110100 & 111100 & 111110 & 011110 & 011100 & 010100 & 010110 \\
110010 & 110000 & 111000 & 111010 & 011010 & 011000 & 010000 & 010010 \\
110011 & 110001 & 111001 & 111011 & 011011 & 011001 & 010001 & 010011 \\
\end{array}
\]
The soft function $r_1(r)$ for QAM-64
Decoding

- The soft functions are calculated for all bits.
- Soft information are accumulated for the entire $N$ systematic information bits and the $2N$ parity bits.
- All the soft information are passed to a Turbo decoder like a MAP.
Simulation Results: TC + QAM 16

Bit error rate $P_e$ vs. $E_b/N_0$ for Turbo coded, RS coded, and Uncoded cases.
Simulation Results: TC + QAM 256, N=2048

Bit error rate $P_e$ vs. $E_b/N_0$

- Uncoded
- Turbo coded
- RS coded
Parity Assisted Decoding (PAD)

- Instead of using soft decoding, we can use hard decision decoding.
- There is penalty for using HD instead of SD.
- PAD attempts to compensate for the loss.
- Trade-off is bandwidth expansion.
PAD Operation

- Two parity bits are generated for every $k$-bit symbol.
- The parity bits are sent using QPSK in a channel different from that used to transmit the QAM symbol.
- They can be sent in the same channel but at alternating time periods.
- The bits are used to assist in making the hard decision during detection.
Choice for PAD Parity Bits

• The parity bits are functions of the bits representing the symbols in the QAM constellation diagram.

• The bits are chosen to be as different as possible from neighboring points.

Let the bits carried by a symbol be $b_0, b_1, \ldots, b_{k-1}$. Then the two parity bits are expressed as

\[
p_0 = f_0 (b_0, b_1, \ldots, b_{k-1}), \quad (1)
\]
\[
p_1 = f_1 (b_0, b_1, \ldots, b_{k-1}), \quad (2)
\]

where $f_0$ and $f_1$ are logic functions.
Choice for PAD Parity Bits ...

Example of received data in QAM-16 square.
Choice for PAD Parity Bits ...

Simulation of 10,000 received symbol when the central point was transmitted.
The first parity bit function $f_0$ truth table for a QAM-16.

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The second parity bit function $f_1$ truth table for a QAM-16.

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Notes on the parity bits

• $p_0$ is more effective in correcting errors than $p_1$.
  • The parity bits can be mapped to protect $p_0$ more than $p_1$.
• The parity bits are more effective when used together.
• It is possible to code the parity bits themselves at the cost of increased BW and complexity.
Decoding Algorithm

• A received QAM symbol is compared to all constellation points.
• \( N \) closest constellation points in Euclidean distance are sorted and stored.
• The parity bits are decoded.
• A hard decision is made to the first point satisfying the first of the following rules:
  1. A decision is made on the first point in the list with matching two parity bits.
  2. The first in the list with a matching \( p_0 \).
  3. The first in the list with a matching \( p_1 \).
  4. The first one in the list (closest to the received symbol).
Varying Relative Energies

- The available energy for transmission is divided unevenly among the parity bits and the information symbol.
- Energy is divided to ensure parity error rate $P_p$ is much less than the expected error for the QAM symbol.
Varying Relative Energies ...

Let energy available for $k$-bit be $E_s$.

$$E_b = \frac{E_s}{k}.$$ 

$E_s$ is distributed amongst the $k$-bit information symbol and two parity bits.

$$E_s = k\tilde{E}_b + 2E_p.$$ 

$\tilde{E}_b$ is the energy per bit of the information symbol and $E_p$ for each parity bit.
Varying Relative Energies ...

\[ E_p = \alpha \tilde{E}_b, \]

for some \( \alpha > 0 \).

\[ kE_b = \tilde{E}_b(k + 2\alpha). \]

\[ Q \left( \sqrt{\frac{2\alpha kE_b}{(k + 2\alpha)N_0}} \right) = \]

\[ 4\beta \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3k^2 E_b}{(k + 2\alpha)(M - 1)N_0}} \right). \]
Varying Relative Energies ...

• Equation (3) can be solved numerically given SNR.
PAD Performance

PAD on QAM-16
PAD Performance

PAD on QAM-64

\[ \frac{E_b}{N_0} \]

\[ \begin{array}{c}
\text{symbol and bit error probability} \\
\text{Uncoded} \\
coded P_{sym} \\
coded P_b \\
\text{Uncoded}
\end{array} \]
Conclusion

• Turbo codes perform close to Shannon’s limit, albeit with complex decoding and long delays.

• TC can be mapped to spectral efficient modulation like QAM using semi-soft information. Performs better than an equivalent RS code.

• PAD is a simple coding technique and is well suited to low computational resources environments.

• PAD can be concatenated with TC and use HD decoding instead of semi-soft information.

• PAD improves on the SNR required to achieve certain BER at the expense of BW expansion.
Suggestions for future research

1. Development of efficient symbol based Turbo codes with reduced complexity decoding algorithms,

2. Development of efficient, even if not optimal, algorithms for decoding binary or symbol based Turbo codes,

3. Improving the block-based Turbo codes and their corresponding decoding algorithms,

4. Finding TC that perform well for short interleaver sizes,

5. Combining TC with MIMO systems and evaluating their performance, and
Acknowledgment

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Acknowledgment ...

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