Wave-particle Duality

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Waves and particles

In classical physics, a particle has a position in space and time, and travels in a straight line (ray) unless acted on by a force, while a wave is a ripple in space-time that carries momentum and energy from one point to another. The ripple may consist of displaced particles such as a surface wave on a pond or a kink in a taut rope. Light waves and gravitational waves, on the other hand, need no medium (carrier particles).

At the microscopic level, physical entities exhibit properties of both particles and waves. An electron microscope, for example, uses the wave nature and short wavelength (5 nanometers) of electrons to obtain high resolution images.

A quantum object is produced as a particle, propagates like a wave, and is detected as a particle with a probability distribution that corresponds to a wave.
Consider a light source $S$ far away from a screen $\Sigma$, producing single photons, one at a time, with isotropic angular distribution. The screen has a slit of width $a$, comparable in size to the wavelength, and the photons that pass through the slit make marks on an observation screen $\Omega$ covered with photographic emulsion — particle-like behavior. Classical particles would be confined to an interval of length $a$, producing a rectangular probability distribution. The photons, however, produce the diffraction pattern due to interference characteristic of all waves as in the following figure.
If we repeat the diffraction experiment with two slits, we obtain a classical probability distribution \( P_C = P_L + P_R \), where \( P_L \) and \( P_R \) are the distributions associated with the left and right slits (each normalized to have integral \( 1/2 \)). The actual distribution is what a continuous light wave would produce as shown on the following page. Feynman called this the central mystery of quantum mechanics. The photons behave as if they are being interfered with by invisible ‘shadow photons’. Accepting the reality of these shadow photons leads to Hugh Everett’s many-worlds or David Deutsch’s multiverse interpretation of quantum mechanics.
Young’s double-slit experiment
Matter waves

The dual nature of material particles was first conjectured by the French physicist Louis de Broglie in the early 1920’s. A photon has momentum \( p = \hbar k = h/\lambda \) and wavelength \( \lambda = h/p \). By the equivalence of mass and energy, the same relationship should hold for material particles. This explains why electrons and photons with the same de Broglie wavelength \( \lambda \) produce the same interference pattern in the double slit experiment.

In 3D the matter wave \( \psi(r, t) \) associated with a particle traveling with momentum \( \mathbf{p} \) in free space is a plane wave of the form

\[
\psi(r, t) = \exp[i(k \cdot r - \omega t)],
\]

where \( k = p/\hbar \) is the wavenumber, and \( \omega = E/\hbar \) is the angular frequency. For relativistic mass \( m, p = mv \) and the total energy is

\[
E = mc^2 = \gamma m_0 c^2, \quad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}.
\]
Quantum mechanics is a framework for construction of physical theories such as QED (Quantum Electrodynamics) and string theory which add additional rules. While quantum mechanics defies intuition, its mathematical formulation enables predictions with a remarkably high degree of accuracy — up to 14 digits of precision in some cases.

To capture the wave-particle duality, we characterize waves by amplitude and particles by position, and then combine the two aspects of nature by associating amplitudes with events. Using Dirac’s notation, the amplitude for a photon generated at position $S$ to arrive at position $x$ on the observation screen is $\langle x | S \rangle$. More generally, $x$ and $S$ are states in a complex Hilbert space, and the amplitude is an inner product and hence a complex number.
Feynman’s Rule: If a particle can produce a certain result by two different routes, the total amplitude for the process is the sum of the amplitudes for the two routes considered separately.

In the double slit experiment with both slits open

\[ \langle x|S \rangle = \langle x|S \rangle_L + \langle x|S \rangle_R, \]

where the subscripts denote amplitudes associated with left and right slits. The rule extends to infinitely many routes. Feynman was not the first person to use the rule, but he formulated it concisely and used it as the basis for his path integral formulation of quantum mechanics. He shared the 1965 Nobel prize in physics with Julian Schwinger and Sin-Itiro Tomonaga.
Amplitude and probability

Unlike classical electrodynamics, our macroscopic devices cannot measure quantum amplitudes. We can only measure probabilities. What is the relationship between the amplitude and the probability of an event? The answer was provided by Max Born in 1926 and earned him the 1954 Nobel prize in physics.

**Born Rule:** \( P(x) = |\langle x|S \rangle|^2 \).

We also have the following.

**Composition Rule:** If a path can be broken up into stages, the total amplitude for traversal of the path is the product of the amplitudes for each of the individual stages.

Combined with Feynman’s rule this gives

\[
\langle x|S \rangle = \langle x|s_L \rangle \langle s_L|S \rangle + \langle x|s_R \rangle \langle s_R|S \rangle,
\]

where \( s_L \) and \( s_R \) denote the left and right slits.
Combining the three rules gives a complete description of the probability distribution $P_Q$ associated with the double-slit experiment.

$$P_Q(x) = |\langle x|S\rangle|^2 = P_L(x) + P_R(x) + I(x),$$

where

$$P_L(x) = |\langle x|s_L\rangle\langle s_L|S\rangle|^2,$$

$$P_R(x) = |\langle x|s_R\rangle\langle s_R|S\rangle|^2,$$

and

$$I(x) = 2 \Re[\langle x|s_L\rangle\langle s_L|S\rangle]^*\langle x|s_R\rangle\langle s_R|S\rangle]$$

is an interference term that would not appear in $P_C(x) = P_L(x) + P_R(x)$. 
There is apparently no way to detect which path a particle takes. If we use a detector to obtain path information the interference pattern vanishes, and whenever we observe interference, we lose path information.

According to the Copenhagen interpretation of quantum mechanics, the particle has no definite position until it is detected, and then the process of measuring its position somehow causes it to appear at the left or right slit (with the appropriate probability) — collapse of the wave function. Objections to this interpretation include the discontinuous jumps when there is an observation, and the difficulty of defining a measuring device. Measurement involves entanglement of the system being measured, the measuring instrument, and the observer. Collapse of the wave function cannot be observed directly, and the boundary between the quantum and classical worlds is ill-defined.