For questions 1-3, indicate yes or no (true/false) for each of the given statements.

1. (20 points) Let \( f(x) = x \) and \( g(x) = x^2 \).
   (a) Is \( f(x) = O(g(x)) \)?
   (b) Is \( f(x) = \Omega(g(x)) \)?
   (c) Is \( f(x) = \Theta(g(x)) \)?

2. (20 points) Let \( f(x) = 2^x \) and \( g(x) = 2^{x/3} \).
   (a) Is \( f(x) = O(g(x)) \)?
   (b) Is \( f(x) = \Omega(g(x)) \)?
   (c) Is \( f(x) = \Theta(g(x)) \)?

3. (20 points) Let \( f(x) = x^{\log 5} \) and \( g(x) = 5^{\log x} \).
   (a) Is \( f(x) = O(g(x)) \)?
   (b) Is \( f(x) = \Omega(g(x)) \)?
   (c) Is \( f(x) = \Theta(g(x)) \)?
4. (20 points) Given the following weighted, directed graph, use the Ford-Fulkerson method for determining the maximum flow.

(a) Using $s$ as the source vertex, and $t$ as the sink vertex, describe each iteration of the algorithm. That is, show or indicate each augmenting path including information about the new flow.

(b) What is the value of the maximum flow?
5. (20 points) You are given a stack data structure that is “dynamic”: when attempting to push onto a stack that is already filled to capacity, it automatically doubles its size before the element is pushed; when the stack is emptied to 1/4 capacity, it automatically halves in size. The stack has two operations: +x, which pushes x onto the stack, and – which pops an element off the stack.

Example: A stack of size four contains four elements. When a fifth element is added, the stack will resize itself to have a capacity of eight.

Example: A stack of size four contains three elements. After two elements are popped so that only one remains, the stack will resize itself to a capacity of two.

Consider the following sequence of operations:

\[\begin{align*}
+ & A + B + C + D - - - + H + J - + L + M + N + P \\
& + Q + R - - - - - - - + W + X + Y + Z
\end{align*}\]

(a) Given an empty stack with capacity one, what is the final size and contents of the stack after all operations have been performed?
6. (20 points) Consider the Diffie-Hellman method for exchanging cryptographic keys with the following parameters: $p = 19, g = 9, a = 5, b = 7$.

(a) At each step, what is private and what is public?

(b) What is the value of the shared secret?

(c) What are the steps needed to break the code?
7. (20 points) Consider the RSA public-key cryptosystem. Given two prime integers $p = 5$ and $q = 7$,

(a) What would be a suitable pair of public/private keys?

(b) Using these keys, what is the encrypted value of 2?