Name: 

For questions 1-3, solve (find an asymptotic \( \Theta \) or \( O \) bound) the recurrence using the “master method.”

1. (20 points) \( T(n) = 8T(n/2) + n/2 \)

2. (20 points) \( T(n) = 7T(n/7) + n \)

3. (20 points) \( T(n) = 5T(n/10) + n \lg n \)
4. (20 points) Given the following weighted graph, use Prim’s algorithm for determining a minimum spanning tree.

(a) Using 0 as the initial vertex, describe each iteration of the algorithm.

(b) What is the weight of the minimum spanning tree?
5. (20 points) Given the following weighted, directed graph, use Dijkstra’s algorithm for single-source shortest paths.

In the space below, indicate each iteration of the algorithm, including which edges are examined and what predecessor and shortest path estimates are being updated.
6. (20 points) You are given a stack data structure that is “dynamic”: when attempting to push onto a stack that is already filled to capacity, it automatically doubles its size before the element is pushed; when the stack is emptied to 1/4 capacity, it automatically halves in size. The stack has two operations: $+x$, which pushes $x$ onto the stack, and $-$ which pops an element off the stack.

*Example:* A stack of size four contains four elements. When a fifth element is added, the stack will resize itself to have a capacity of eight.

*Example:* A stack of size four contains three elements. After two elements are popped so that only one remains, the stack will resize itself to a capacity of two.

Consider the following sequence of operations:

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+1 +2 +3 +4 - - - +5 +6 - +7 +8 +9 +10
+11 +12 - - - - - - +13 +14 +15 +16
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(a) Given an empty stack with capacity one, what is the final size and contents of the stack after all operations have been performed?

(b) Prove that the amortized average of cost of the push and pop operations is $O(1)$. 

7. (20 points) Consider a skip list data structure with the following property for the insertion operation: after the element is added to the “bottom” list, a (fair) coin is tossed and if it comes up heads that element is added to the next level up, and flipped again. This is done repeatedly until the coin comes up tails. For the following sequence of elements

\[22 \ 1 \ 8 \ 13 \ 2 \ 4 \ 7 \ 17 \ 20 \ 23 \ 14\]

and the following coin flip outcomes

\[T \ H \ T \ H \ T \ H \ H \ H \ T \ H \ T \ H \ H \ T \ H \ T \ H \ H \ T \ T\]

draw a picture of the final skip list after all elements have been inserted.