1. List the output of the following program (which includes both animalHierarchy.h and main.cpp).

```cpp
using namespace std;
#include <string>

class Animals{
    private:
        string name;
    public:
        Animals(string n) { name = n; }
        string getName() { return name; }
        virtual string action() = 0;
};
class Mammals : public Animals{
    public:
        Mammals(string n): Animals(n) {} 
        string action() { return getName() + " says Bye!"; }
};
class Birds : public Animals {
    public:
        Birds(string n): Animals(n) {} 
        string action() { return getName() + " says: Hello!"; }
};
class Dog : public Mammals{
    public:
        Dog(string n): Mammals(n) {} 
        string action() { return getName() + " says: Good Morning!"; }
};
class Penguin : public Birds {
    public:
        Penguin(string n): Birds(n) {} 
        string action() { return getName() + " says: Good Night!"; }
};
```
Listing 2: main.cpp

```cpp
using namespace std;

#include <iostream>
#include "animalHierarchy.h"

int main()
{
    Mammals m = Mammals("Bruno");
    Dog d1 = Dog("Bruno");
    Dog d2(m.getName());
    Birds b = Birds("Tweety");
    Birds p1 = Penguin(b.getName());
    Penguin p2 = Penguin(b.getName());

    cout << m.action() << endl;
    cout << d1.action() << endl;
    cout << d2.action() << endl;
    cout << b.action() << endl;
    cout << p1.action() << endl;
    cout << p2.action() << endl;

    return 0;
}
```
Recall that mathematical induction is a technique for proving a certain statement or property about the set (or a subset) of natural numbers. To do so, first a basis case of the statement is proven: this is where we show that the statement is true for a particular value of \( n \). Then, the inductive step is proven: here we assume the statement is true for some arbitrary value of \( n \) (this is called the induction hypothesis) and use that fact to show that the statement is true for \( n + 1 \).

2. Prove the following statement using induction (\( n \) is a natural number).

\[
\sum_{i=1}^{n} 2i - 1 = n^2
\]

**HINT:** If you are unfamiliar with \( \Sigma \) notation, the following statement is equivalent:

\[
1 + 3 + 5 + 7 + 9 + \cdots + (2(n - 2) - 1) + (2(n - 1) - 1) + (2n - 1) = n^2.
\]

In other words, the statement you are being asked to prove is: “the sum of the first \( n \) odd numbers is equal to \( n^2 \).
3. Consider the expression \((P \lor (Q \land R)) \rightarrow ((P \lor Q) \land R)\). Note that \(\lor\) is the symbol for disjunction (logical OR), \(\land\) is the symbol for conjunction (logical AND) \(\neg\) is the symbol for negation (NOT) and \(\rightarrow\) is the symbol for implication.

(a) Write the truth table.

(b) Is the expression a tautology?
4. Write a Haskell or C/C++ program that solves the following cryptarithmetic puzzle, where distinct letters should correspond to distinct digits from 0 to 9.

\[ \text{TOD} + \text{TOD} + \text{TOD} = \text{GOOD} \]